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Natural convection in enclosures with variable fluid properties

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Abstract *The generalized integral transform technique (GITT) is an hybrid numerical-analytical method that has been successfully applied in convection-diffusion problems, where the original potentials are replaced by eigenexpansion series, and the system of partial differential equations is transformed into a finite system of ordinary differential equations, allowing to obtain an error controlled solution without any kind of grid generation. This paper aims at the application of GITT to the transient version of the classical differentially heated square cavity problem, considering fluid properties as functions of temperature. Comparing results to some previously reported data for constant fluid properties validates the computational procedure. The solution for variable fluid properties with Boussinesq approximation is presented for several values of inclinations, at Rayleigh number of 10^3 and a Prandtl number of 0.71, demonstrating GITT capability of capturing circulating cells formation and evolution at a low Rayleigh number. New correlations for leaning angle and aspect ratio are presented.*

Nomenclature

a	= cavity length	Nu	= Nusselt number
b	= cavity height	P	= pressure
C_p	= specific heat at constant pressure, dimensional and dimensionless	Pr	= Prandtl number
		r	= aspect ratio
		Ra	= Rayleigh number
g	= gravity acceleration	Re	= Reynolds number
k	= thermal conductivity, dimensional and dimensionless	t	= time
		T	= temperature
		u	= longitudinal velocity

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component, dimensionless
v = transversal velocity component, dimensionless
y = transversal coordinate
$Y_i(x)$ or $Y_j(y)$ = normalized eigenfunction of order i/j for the streamfunction expansion
x = longitudinal coordinate, dimensionless

Greek symbols
$\bar{\phi}_i(x)$ = normalized eigenfunction of order i for the temperature expansion in the x-direction
μ = fluid absolute viscosity, dimensional and dimensionless
v = fluid kinematic viscosity
ψ = streamfunction
$\Psi_{ij}(t)$ = transformed streamfunction
$\bar{\Gamma}_j(y)$ = normalized eigenfunction of order j for the temperature expansion in the y-direction
γ = leaning angle
θ_0 = dimensionless temperature difference
$\Theta_{ij}(t)$ = transformed temperature
ρ = fluid density, dimensionless

Subscripts and superscripts
* = relative to dimensional quantities ($x, y, u, v, T, P, \rho, Q$)
0 = relative to properties estimated at initial temperature

Introduction

Processes of inner natural convection, besides having a vast number of applications in different areas of engineering, also represent a good test for numerical methods, due to strong non-linear coupling among equations of movement and energy. The classic problem of square cavity (lid-driven flow) with sides warmed at different temperatures has been used as a test to compare different numerical methods, and represents a challenge when the intensity of the process causes instabilities, as in the case of high Rayleigh numbers (Leal, 1996).

Most studies have been done using the Boussinesq approximation (1903) to simplify the solution of practical problems. The approximation is based on two principles: variation of fluid density is important only in the buoyancy term and other physical properties are considered constant.

Such hypothesis not always reproduces the physical phenomena with the desired precision, depending on the imposed temperature difference between the walls (Zhong *et al.*, 1985). A non-Boussinesq model considering the variation of properties taken as a whole and individually and still introducing geometric variables, like angle variation in the problem of square cavity, could reproduce more realistic physical effect in the cavities. The establishment of reliable benchmark results in transient-state is of major interest in allowing critical comparisons among different scheme variants and computational implementation strategies.

The influence of the fluid properties variation with temperature has appeared as an important aspect to be analyzed in this class of problems. The well-known Boussinesq approximation has been extensively employed, but very little research has been undertaken to inspect the influence of variable thermophysical properties in the flow structure, with or without the Boussinesq simplification. We can mention others researches to our present purposes, such as, Bergles (1983) who presented a correlation formula to compute the influence of each property (viscosity and conductivity) for forced convection in tubes, considering incompressible flow. Gray and Giorgini (1976) studied the limits of application of the Boussinesq approximation to external flows of water and air, using two orders of approximation: strict and extended. They also presented graphics to indicate accurate limits of those hypotheses. The stability and the limits of the Boussinesq approximation were also the subject in the works of Graham (1975) and Spradley and Churchill (1975), both using the finite difference method to compute the lid-driven cavity problem for a compressible fluid with variable properties. Suslov and Paolucci (1995) reproduced and extended the results of these works, aimed at finding the critical Rayleigh number, and showing the presence of two regimen of instability, one of them due to the non-Boussinesq effects. Yu *et al.* (1996) tried to present a benchmark for the compressible problem (the lid-driven cavity), using finite element analysis, and handling the common limitations of this method when applied to low-Mach number compressible flows. Finally, Zhong *et al.* (1985) revised the work of Graham (1975), centering their study in the validity of the Boussinesq approximation. They found a more strict limit than the one presented by Gray and Giorgini (1976), despite the good agreement achieved for Nusselt number calculations.

The generalized integral transform technique (GITT) is a relatively recent method for the solution of partial differential equations (Cotta and e Mikhailov, 1998), and has shown itself as an alternative to the purely discrete methods. Its numerical-analytical hybrid character allows the automatic error control during the solution of the equations. This avoids the need for many executions of the computational code for convergence, dispenses grid generation and allows an easy extension to a larger number of dimensions involved in the problem.

In GITT, the need to find an exact integral transform for the problem is relieved by an auxiliary eigenvalue problem, most representative as possible from the original problem. The original potentials are represented by an infinite summation of the eigenfunctions, obtained from the auxiliary problem and the transformed potentials. When the transform is applied, one obtains an infinite coupled ordinary differential system of equations that is truncated in a sufficient order to reach the desired accuracy. The system is solved through well-established algorithms, that have automatic error control and are available in libraries of scientific routines such as DIVPAG (IMSL Library, 1989).

In this work, the problem of the cavity with differential heating is solved by GITT for different leaning angles and aspect ratios, using air as working fluid and considering all the properties as functions of the temperature. The method of solution is validated comparing the results for a square cavity horizontally arranged, with constant properties (and available in literature), in a range of temperature where Boussinesq approximation is valid. One of the aims was to verify the GITT capability of capturing more complex characteristics of the phenomena, such as recirculations and obtaining a correlation for the flow behavior as a function of the cavity leaning angle.

From the results, it was possible to prove the physical consistence of the method at low Rayleigh numbers. In addition, the results prove that GITT was capable of capturing recirculations, even of low intensity, and obtaining a correlation for the convection heat transfer for variable properties, providing, for the case simulated, Pr = 0.7 and Ra = 10^3, an averaged Nusselt number as a function of the cavity leaning angle.

Physical problem

The problem to be analyzed consists of a rectangular cavity, filled with air (Prandtl number: Pr = 0.71), with sides a and b and leaning angle γ, which the parallel walls to the x axis are thermally isolated and the walls parallel to the y axis are at constant and uniform temperatures, as shown in Figure 1.

The simplifying hypotheses assumed for the current problem are: two-dimensional, laminar flow of a Newtonian fluid with constant density, except in the thermal-driven force term, impermeable wall and no-slip condition in the walls.

The cavity dimensions are normalized by the sides a and b. From usual applied dimensionless processes, the equations in stream function formulation become:

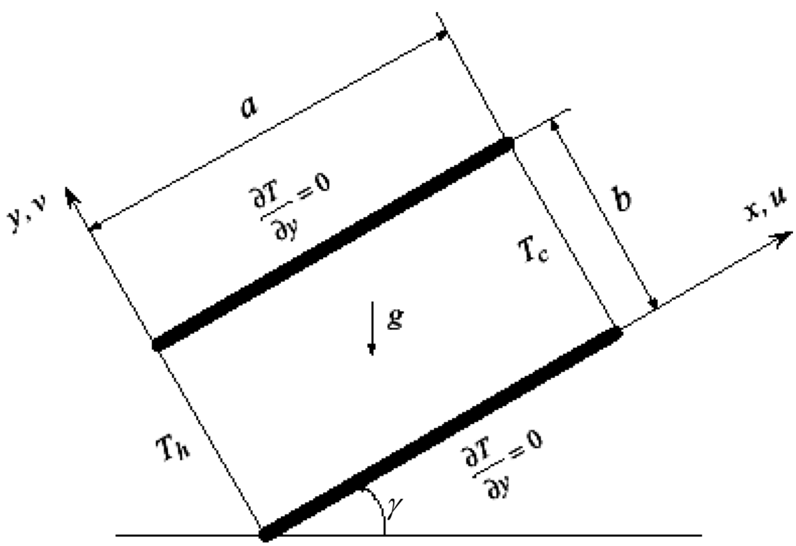


Figure 1.
Cavity geometry

$$\nabla^2\left(\frac{\partial\Psi}{\partial t}\right)+\frac{\partial\Psi}{\partial y}\nabla^2\left(\frac{\partial\Psi}{\partial x}\right)-\frac{\partial\Psi}{\partial x}\nabla^2\left(\frac{\partial\Psi}{\partial y}\right)\frac{\partial\Psi}{\partial y}$$

$$=r\mathrm{Pr}_0\left[\frac{\mu}{r}\nabla^4\Psi+2r\frac{\partial\mu}{\partial y}\nabla^2\left(\frac{\partial\Psi}{\partial y}\right)+\frac{2}{r}\frac{\partial\mu}{\partial x}\nabla^2\left(\frac{\partial\Psi}{\partial x}\right)\right.$$

$$+4r\frac{\partial^2\mu}{\partial x\,\partial y}\frac{\partial^2\Psi}{\partial x\,\partial y}+\left(r^2\frac{\partial^2\mu}{\partial y^2}-\frac{\partial^2\mu}{\partial x^2}\right)\left(r\frac{\partial^2\Psi}{\partial y^2}-\frac{1}{r}\frac{\partial^2\Psi}{\partial x^2}\right) \quad (1)$$

$$\left.-\mathrm{Ra}_0\mathrm{Pr}_0\frac{\partial T}{\partial x}\cos\gamma+r\mathrm{Ra}_0\mathrm{Pr}_0\frac{\partial T}{\partial y}\sin\gamma\right]$$

$$\frac{\partial T}{\partial t}+\frac{\partial\Psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial T}{\partial y}=\frac{1}{\rho_0 C_{\mathrm{p}_0}}\left(k\nabla^2 T+\frac{\partial k}{\partial x}\frac{\partial T}{\partial x}+r^2\frac{\partial k}{\partial y}\frac{\partial T}{\partial y}\right) \quad (2)$$

where $r=a/b$ is the aspect ratio. The boundary conditions for the problem in dimensionless form are:

$$T=1,\quad \psi=\psi_y=0\quad \text{at}\quad x=0 \quad (3a)$$

$$T=0,\quad \psi=\psi_y=0\quad \text{at}\quad x=1 \quad (3b)$$

$$T_y=0,\quad \psi=\psi_x=0\quad \text{at}\quad y=0 \quad (3c)$$

$$T_y=0,\quad \psi=\psi_x=0\quad \text{at}\quad y=1 \quad (3d)$$

The initial conditions are:

$$T(x,y,0)=0 \quad (3e)$$

$$\psi(x,y,0)=0 \quad (3f)$$

where the reference Rayleigh and Prandtl numbers are defined as:

$$\mathrm{Ra}_0=\frac{g\beta(T_{\mathrm{h}}-T_{\mathrm{c}})a^3}{\alpha_0\mu_0} \quad (4a)$$

$$\mathrm{Pr}_0=\frac{\mu_0 C_{\mathrm{p}_0}}{k_0} \quad (4b)$$

and the remaining dimensionless variables are given by:

$$\psi = \frac{\psi_*}{\alpha_0}; \tag{4c}$$

$$t = \frac{\alpha_0}{a^2} t_*; \tag{4d}$$

$$x = \frac{x_*}{a}; \tag{4e}$$

$$y = \frac{y_*}{b}; \tag{4f}$$

$$T^* = \frac{T_* - T_c}{T_h - T_c}; \tag{4g}$$

$$\mu = \frac{\mu_*}{\mu_0}; \tag{4h}$$

$$k = \frac{k_*}{k_0}; \tag{4i}$$

$$C_p = \frac{C_{p*}}{C_{p0}} \tag{4j}$$

where the subscript "*" identifies the dimensional variables, the subscript "0" denotes the property estimate at the initial temperature, except for the reference properties, which are estimated at the film temperature, α is the fluid thermal diffusivity, μ is the variable kinematic viscosity, k is the variable thermal conductivity, C_p is the variable specific heat, T_h is the hot wall temperature, T_c is the cold wall temperature, g is the gravity acceleration and β is the fluid volumetric expansion coefficient.

The boundary conditions of temperature are homogenized through the filter $T(x,y,t)(\text{filtered}) = T(x,y,t) + T_f(x)$, where $T_f(x) = 1 - x$.

Numerical solution through integral transformation

In order to solve convective-diffusive problems by GITT, the steps to be followed are to define and solve auxiliary problem to obtain the eigenfunctions, eigenvalues and norms, to develop the direct and inverse transforms pairs, achieve the integral transform of the partial differential system in a coupled ordinary differential system and obtain the original potentials through inversion formula.

The auxiliary problems for each variable are chosen from Sturm-Liouville eigenvalue problems of fourth and second-order with homogeneous boundary conditions for streamfunction and temperature, respectively (Leal, 1996):

The transformed potentials and inversion formulas are given by:

$$\Psi_{i,j}(t) = \int_0^1 \int_0^1 Y_i(x) Y_j(y) \psi(x,y,t)\, \mathrm{d}y\, \mathrm{d}x \tag{5a}$$

$$\Theta_{i,j}(t) = \int_0^1 \int_0^1 \bar{\phi}_i(x) \bar{\Gamma}_j(y) T(x,y,t)\, \mathrm{d}y\, \mathrm{d}x \tag{5b}$$

$$\psi(x,y,t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} Y_i(x) Y_j(y) \Psi_{i,j}(t) \tag{6a}$$

$$T(x,y,t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \bar{\phi}_i(x) \bar{\Gamma}_j(y) \Theta_{i,j}(t) \tag{6b}$$

The ordinary differential equations system is obtained from the integral transform of equations (1) and (2), after application of integral operators, replacing the inversion formulas (equation (6)), and taking advantage of the orthogonality property of eigenfunctions (Mikhailov and Özisik, 1984), giving:

$$\sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \left[\int_0^1 Y_i(x) Y_k''(x) \mathrm{d}x \int_0^1 Y_j(y) Y_l(y) \mathrm{d}y + \int_0^1 Y_i(x) Y_k(x) \mathrm{d}x \int_0^1 Y_j(y) Y_l''(y) \mathrm{d}y \right]$$

$$\frac{\mathrm{d}\Psi_{kl}(t)}{\mathrm{d}t} = + \int_0^1 \int_0^1 Y_i(x) Y_j(x) \left\{ \frac{\partial \Psi}{\partial y} \nabla^2 \left(\frac{\partial \Psi}{\partial x} \right) - \frac{\partial \Psi}{\partial x} \nabla^2 \left(\frac{\partial \Psi}{\partial y} \right) \frac{\partial \Psi}{\partial y} \right.$$

$$- r\mathrm{Pr}_0 \left[\frac{\mu}{r} \nabla^4 \Psi + 2r \frac{\partial \mu}{\partial y} \nabla^2 \left(\frac{\partial \Psi}{\partial y} \right) \right] - r\mathrm{Pr}_0 \left[\frac{2}{r} \frac{\partial \mu}{\partial x} \nabla^2 \left(\frac{\partial \Psi}{\partial x} \right) + 4r \frac{\partial^2 \mu}{\partial x \partial y} \frac{\partial^2 \Psi}{\partial x \partial y} \right. \tag{7}$$

$$\left. \left. + \left(r^2 \frac{\partial^2 \mu}{\partial y^2} - \frac{\partial^2 \mu}{\partial x^2} \right) \left(r \frac{\partial^2 \Psi}{\partial y^2} - \frac{1}{r} \frac{\partial^2 \Psi}{\partial x^2} \right) - \mathrm{Ra}_0 \mathrm{Pr}_0 \frac{\partial T}{\partial x} \cos\gamma + r\mathrm{Ra}_0 \mathrm{Pr}_0 \frac{\partial T}{\partial y} \sin\gamma \right] \right\}$$

$$\frac{d\Theta_{i,j}(t)}{dt} = \int_0^1 \int_0^1 \bar{\phi}_i(x)\bar{\Gamma}_i(y)\left\{\frac{\partial\Psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial T}{\partial y} - \frac{1}{\rho_0 C_{p_0}}\left(k\nabla^2 T + \frac{\partial k}{\partial x}\frac{\partial T}{\partial x} + r^2\frac{\partial k}{\partial y}\frac{\partial T}{\partial y}\right) + \frac{dT_h}{dt}\right\} dy\, dx \tag{8}$$

Equations (7) and (8), can be rewritten in a compact form as:

$$\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} E_{ijkl}\frac{d\Psi_{kl}(t)}{dt} = F_{ij}(t) \tag{9}$$

$$\frac{d\Theta_{i,j}(t)}{dt} = G_{ij}(t) \tag{10}$$

The transformed initial conditions are obtained by operating the following original initial conditions:

$$\Psi_{ij}(t=0) = 0 \tag{11a}$$

$$\Theta_{i,j}(t=0) = \int_0^1 \int_0^1 \bar{\phi}_i(x)\bar{\Gamma}_j(y)(x-1)\, dy\, dx \tag{11b}$$

Equations (9) and (10) constitute an ordinary differential system of stiff type, where the transformed potentials have the decaying rates very distinguished from each other. For numerical solution of this system, the subroutine DIVIPAG (IMSL Library, 1989), which allows automatic control of local error, keeping it inside the desired tolerance, was applied. The expansions are truncated in a finite number of terms, NC and NT, for stream function and temperature, respectively, resulting in a system of coupled differential equations. The number of equations obtained after transformation in the two directions are $NC^2 + NT^2$.

At each step of time, the coefficients F_{ij} and G_{ij} must be numerically integrated. For reduction of computational costs, it was employed in the algorithm, developed by Machado (1999), that uses the integration technique of Gauss Quadrature (and made the solution feasible).

Convergence analysis

In order to validate the code, the same number of 120 terms was used for stream function (NC) and temperature (NT). The precision used in the integration of the system and in the solution of the ordinary differential equations system was 10^{-4}. Initially, the code was validated comparing the results obtained for low differences of temperature (inside validity limit of Boussinesq approximation), with those obtained through GITT for a fluid with constant properties (Leal, 1996), considering $\gamma = 0$ and $r = 1$. The limit of validity for the Boussinesq approximation was proposed by Zhong *et al.* (1985) as:

$$\theta_0 = 0.0244\,\mathrm{Ra}^{0.243} \tag{12a}$$

where

$$\theta_0 = (T_h - T_c)/T_c \tag{12b}$$

The limit for $\mathrm{Ra} = 10^3$ is $\theta_0 = 0.13$ In Table I, results for variable fluid properties and $\theta_0 = 0.0101$ show excellent agreement with the results for constant properties, until the third significant algorithm, for stream function and temperature.

The convergence analysis was made for the most critical case considered, i.e. aspect ratio 10 and leaning angle 45°. Table II shows the results for stream function and temperature in function of the number of integration points used in the Gauss quadrature – NF – for a non-Boussinesq case, where $\theta_0 = 0.5$ (out of the limit of application of the Boussinesq hypothesis), with $\mathrm{Ra} = 10^3$ at $t = 0.005$. It was verified that NF = 30 was enough to make the convergence reach the desired accuracy.

In Table III, the convergence of the streamfunction and temperature are shown with simultaneous variation of truncation order of each series, NC and NT, in the case discussed earlier. From the results, 120 terms for NC and NT were considered enough to reach the desired accuracy, and this number of terms has been employed in the rest of this work.

x	*y*	10	20	30	40	Leal (1996)
Stream function × NF (NC and NT = 120)						
0.1	0.1	−0.0449	−0.0393	−0.0393	−0.0393	−0.0393
0.1	0.3	−0.0774	−0.0966	−0.0967	−0.0967	−0.0966
0.1	0.9	−0.0449	−0.0396	−0.0395	−0.0395	−0.0395
0.3	0.1	−0.0011	−0.0153	−0.0153	−0.0153	−0.0153
0.3	0.3	−0.0117	−0.0755	−0.0755	−0.0755	−0.0757
0.3	0.9	−0.0017	−0.0154	−0.0154	−0.0154	−0.0155
0.9	0.1	−0.0000	−0.0011	−0.0011	−0.0011	−0.0011
0.9	0.3	−0.0066	−0.0062	−0.0062	−0.0062	−0.0062
0.9	0.9	−0.0001	−0.0011	−0.0011	−0.0011	−0.0011
Temperature × NF (NC and NT = 120)						
0.1	0.1	0.4760	0.4767	0.4765	0.4765	0.4783
0.1	0.3	0.4124	0.4819	0.4821	0.4819	0.4874
0.1	0.9	0.4900	0.4874	0.4875	0.4874	0.4891
0.3	0.1	0.0203	0.0004	0.0004	0.0004	0.0004
0.3	0.3	−0.0046	0.0005	0.0005	0.0005	0.0005
0.3	0.9	0.0208	0.0005	0.0005	0.0005	0.0005
0.9	0.1	−0.0042	0.0000	0.0000	0.0000	0.0000
0.9	0.3	0.0261	0.0000	0.0000	0.0000	0.0000
0.9	0.9	−0.0044	0.0000	0.0000	0.0000	0.0000

Table I. Results of present work compared to Leal (1996) for a square horizontal cavity, within the limit of Boussinesq hypothesis ($\theta_0 = 0.0101$), considering $\mathrm{Ra} = 10^3$ at $t = 0.01$

Table II. Convergence for the variation of the number of terms used in the Gauss Quadrature integration – NF, for the case of $r = 10$, $\gamma = 45°$, with $Ra = 10^3$ and $\theta_0 = 0.5$, at $t = 0.005$

x	y	10	20	30
Stream function $\times$ *NF (NC and NT = 120)*				
0.1	0.1	0.00051	−0.00116	−0.00116
0.1	0.5	0.00498	−0.00868	−0.00866
0.1	0.9	0.00046	−0.00116	−0.0116
0.5	0.1	0.00228	0.00003	0.00003
0.5	0.5	0.01403	0.00021	0.00018
0.5	0.9	0.00226	0.00003	0.00003
0.9	0.1	0.00018	0.00000	0.00000
Temperature $\times$ *NF (NC and NT = 120)*				
0.1	0.1	0.33058	−0.38636	−0.38619
0.1	0.5	0.28667	−0.38643	−0.38627
0.1	0.9	0.33052	−0.38651	−0.38634
0.5	0.1	−0.01376	−0.00042	−0.00038
0.5	0.5	−0.02056	−0.00042	−0.00038
0.5	0.9	−0.01376	−0.00042	−0.00038
0.9	0.1	0.02558	0.00015	0.00026

Table III. Convergence for the variation of the truncation order of the expansions (NC/NT), for the case of $r = 10$, $\gamma = 45°$, with $Ra = 10^3$ and $\theta_0 = 0.5$ at $t = 0.01$, using NF = 40

x	y	60/60	80/80	100/100	120/120
Stream function					
0.1	0.1	−0.02437	−0.02540	−0.02552	−0.02566
0.1	0.3	−0.06500	−0.06600	−0.06596	−0.06570
0.1	0.9	−0.02444	−0.02544	−0.02556	−0.02569
0.3	0.1	−0.01195	−0.01231	−0.01235	−0.01222
0.3	0.3	−0.05939	−0.05984	−0.05960	−0.05969
0.3	0.9	−0.1205	−0.1240	−0.1244	−0.1232
0.9	0.1	−0.00082	−0.00096	−0.00084	−0.00088
Temperature					
0.1	0.1	0.54481	0.54774	0.54760	0.54767
0.1	0.3	0.55817	0.55122	0.55113	0.55124
0.1	0.9	0.55155	0.55470	0.55466	0.55482
0.3	0.1	−0.00010	0.00091	0.00054	0.00057
0.3	0.3	−0.00003	0.00089	0.00059	0.00061
0.3	0.9	0.00005	0.00088	0.00064	0.00064
0.9	0.1	−0.00020	0.00005	−0.00005	0.00007

Results

For all simulations, $Ra = 10^3$ and $\theta_0 = 0.5$ were considered. The first aspect analyzed was the sensitivity of flow to the individual variation of each property, compared with the cases of fluid with constant properties and all properties varying for the case $\gamma = 0$ and $r = 1$. The functions employed to represent the fluid physical properties variation with temperature, provided by Zhong *et al.* (1985) in dimensionless form, are written as:

$$k = \frac{2.6482 \times 10^{-3} T^{\frac{3}{2}}}{T + 245.4 \times 10^{-\frac{12}{T}}} \quad (13a)$$

$$C_{\mathrm{p}} = 9898.24 - 0.3316T + 0.2025 \times 10^{-3} T^2 \quad (13b)$$

$$\mu = \frac{14.58 \times 10^{-7} T^{\frac{3}{2}}}{110.4 + T} \quad (13c)$$

Analyzing the Zong's equations between temperatures of 300 and 500 K, we can see an increase of 50 percent approximately in the thermal conductivity, 40 percent for the viscosity and a decrease of 4 percent for the specific heat, as well. In Figure 2, it is possible to observe that the conductivity increases with temperature and, there will be an increase in the thermal diffusivity, raising the thermal boundary layer, resulting in a larger displacement of the stream functions compared to the case of constant properties. The variation of viscosity gives an opposite effect, since the viscosity increases with temperature inducing an increase in the momentum dissipation and consequently decreasing the displacement intensity of the stream function. However, with lesser intensity than the conductivity, specific heat has little effect over the behavior of the stream functions compared with the constant properties. Nevertheless, we must take into account that the Rayleigh number is low. At higher values of Rayleigh number, the influence of specific heat tends to intensify (Leal *et al.*, 2000).

Inclined cavities have the same behavior of the non-inclined cavities for variable properties, only intensifying and not effecting the function of the gravitational force. Figure 3, shows the results for all variable properties and different leaning angles, where the ratio a/b is equal to 1.0 in $t = 0.02$. At a lesser time, the stream function shows higher intensity of the streamlines, which has their center of rotation in $y = 0.5$ and $x < 0.5$. In this time interval, the recirculation is more intense near the warmed wall. This phenomenon is known by other researchers and, could be captured by the GITT technique. Once the time increases, this center is displaced to $y = 0.5$ and $x = 0.5$. We note a higher intensity of streamlines for the angles $\gamma = 15°$ and 0°, compared to the higher angles. The temperature profiles in this case present a more evident effect of the gravitational term, as shown in Figure 4. For higher inclination angles, the isotherms are almost linear.

Figure 5 compares the streamlines for cases $\gamma = 15°$, $r = 1$ and $\gamma = 45°$, $r = 10$. A small influence over the flow at low leaning angles verifies Figure 5(a). In Figure 5(b), the most critical case simulated is shown, and a substantial alteration in the streamlines can be observed, as the vanishing center of rotation.

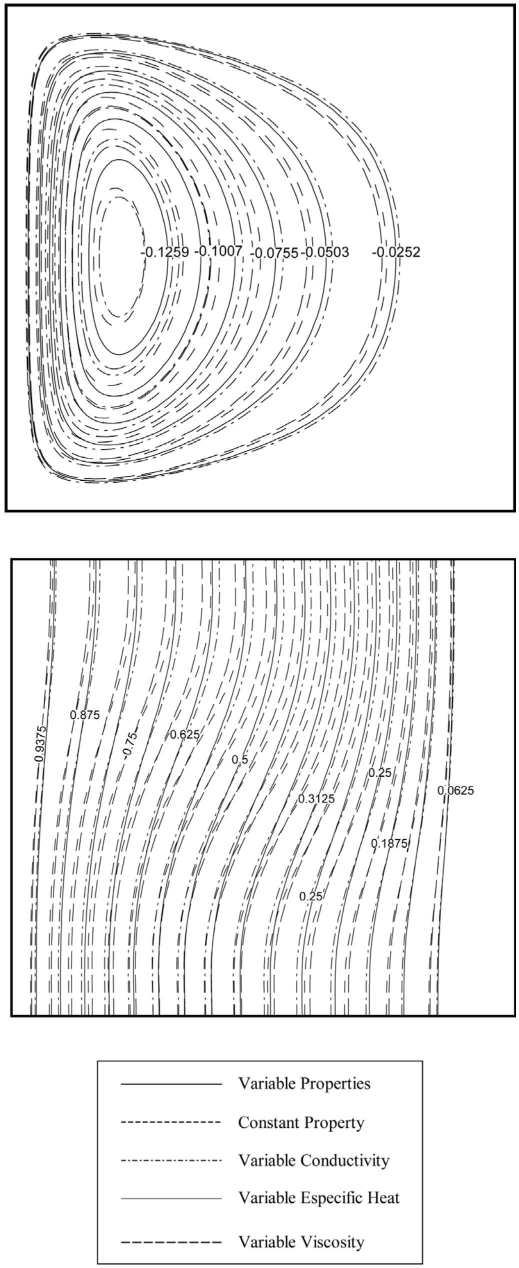


Figure 2.
Streamlines and isotherms for $t = 0.5$

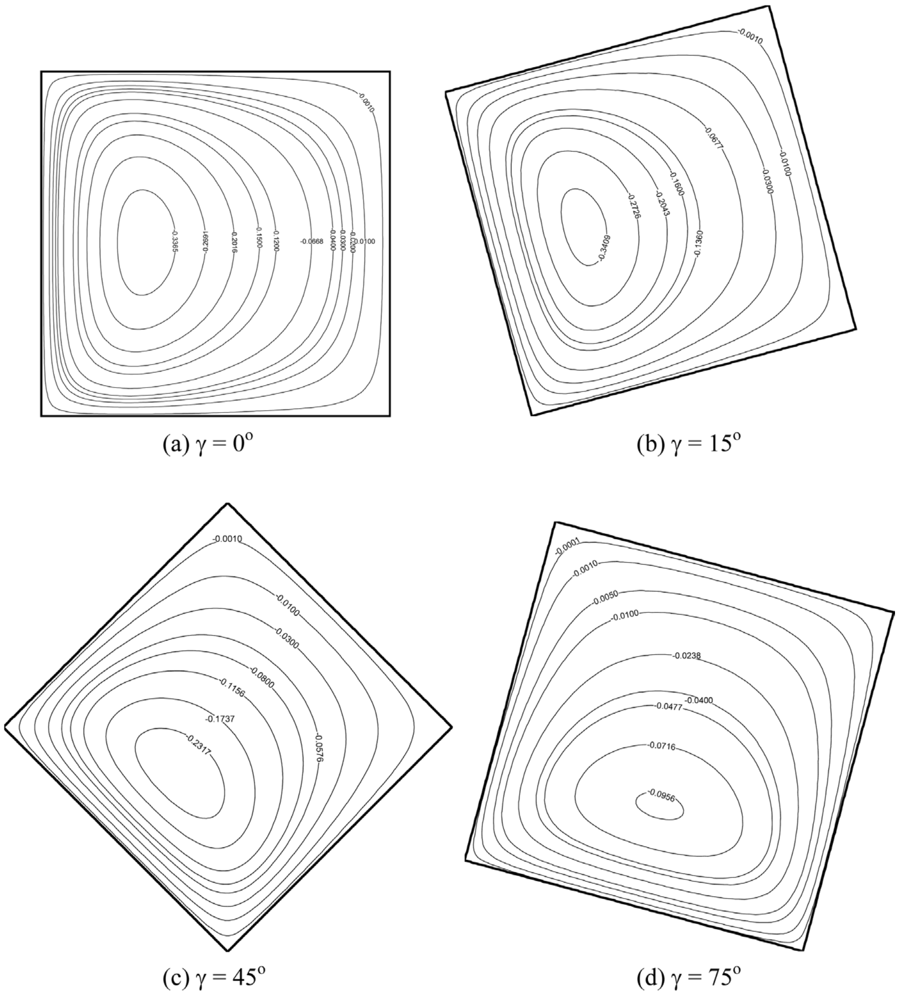


Figure 3.
Streamlines for different leaning angles and $t = 0.02$

Based on the results obtained for the mean Nusselt, using $\mathrm{Ra} = 10^3$ and $\mathrm{Pr} = 0.7$, we can correlate this for an inclined cavity of aspect ratio equal to 1, a polynomial relation that was found as follows:

$$\mathrm{Nu_m} = 0.9165 + 0.00026\gamma - 1.24 \times 10^{-5}\gamma^2 \tag{14}$$

The values for average Nu according to Figure 6, show a high exchange of convective heat at angles between 0 and 30°. It presents a maximum value for the average Nusselt number at an angle of approximately 15°. The influence of the heat conductivity on the heat transfer process as the leaning angle increases is also evident.

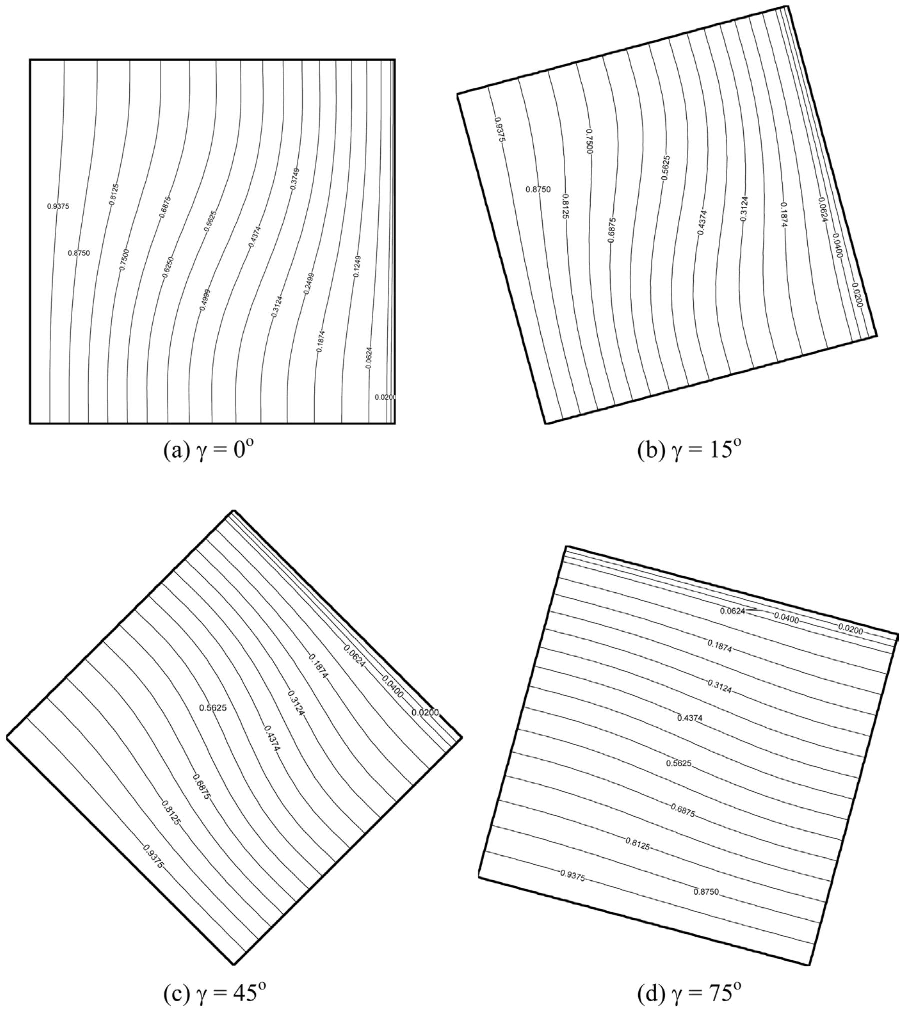
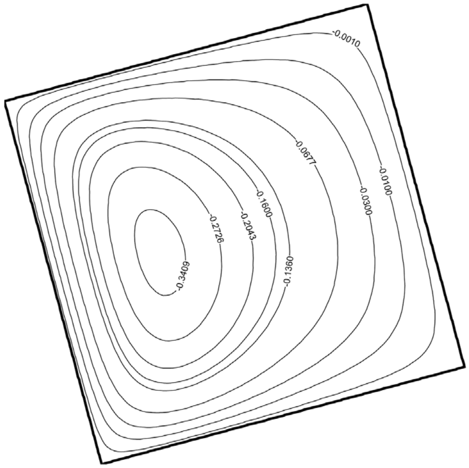


Figure 4.
Isotherms for different leaning angles and $t = 0.5$

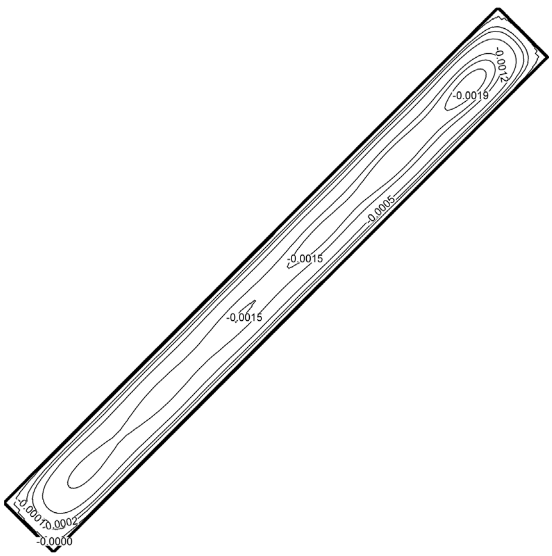
Figure 7 shows the variation of the mean Nusselt number in an horizontal cavity, as a function of the aspect ratio. The graphic shows an exponential decrease, which can be represented by the correlation:

$$\mathrm{Nu_m} = \frac{0.86078}{(1 - \mathrm{e}^{-2.88r})} \tag{15}$$

Equation (15) provides an error smaller than 0.5 percent, that vanishes as r increases. Such behavior is due to the lowering of the cavity height, diminishing of the buoyancy effect and consequently decreasing the convective heat transfer.



(a) $\gamma = 15^{o}$ and r = 1 , t = 0. 02



(b) $\gamma = 45^{o}$ and r = 10, t = 0.5

Figure 5.
Streamlines in two distinct cases

Conclusions

In this work, the natural convection process inside an inclined rectangular cavity was studied, with air as the working fluid considering variable thermo-physical properties. The GITT was used to solve the governing equations.

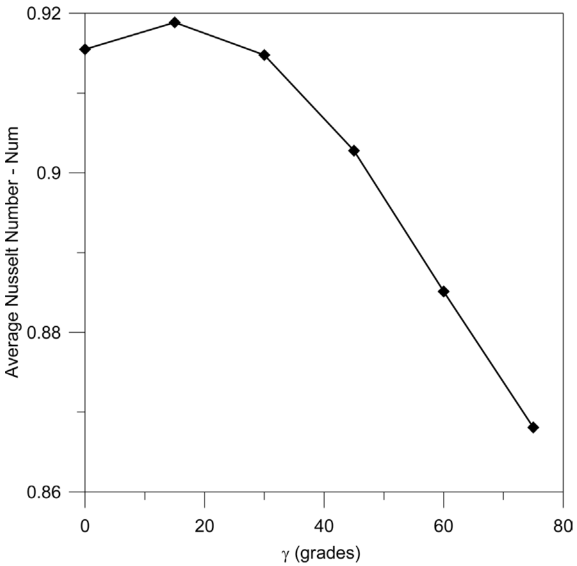


Figure 6.
Average Nusselt number as a function of the leaning angle

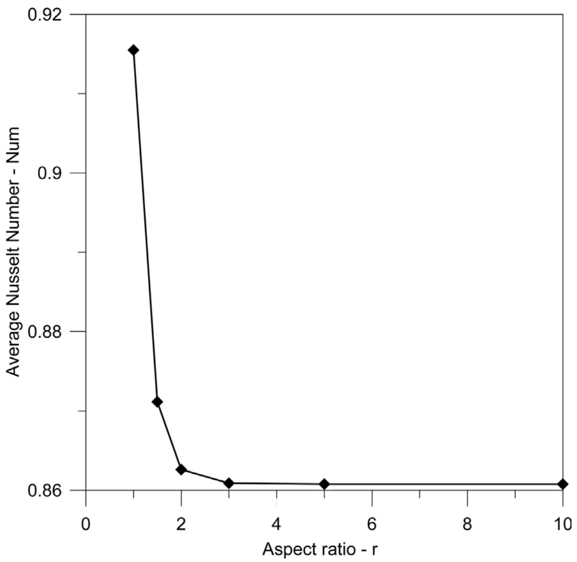


Figure 7.
Average Nusselt number as a function of the aspect ratio

Initially, the method was validated by comparing the results of a problem inside the validity limits of Boussinesq approximation, to a fluid with constant properties. In a second instance, a convergence analysis was made, in order to establish a truncation order for the streamlines and temperature expansions and the number of Gauss points used in numerical integration of coefficients.

For a particular Rayleigh number (10^3), the flow sensitivity was observed by varying each property, using their correlations with temperature, and simultaneously varying the leaning angle for all properties.

The results allowed us to conclude that thermal conductivity and dynamic viscosity are the properties most sensitive to temperature variations, and also have influence on heat flux as much as on the local flow temperatures and streamlines. For a cavity with aspect ratio equal to 1, it was possible to determine an ideal angle for convective heat transfer in cavities, and to obtain correlations for the mean Nusselt number in function of the leaning angle and aspect ratio, where an asymptotic value was found as the aspect ratio increases.

It was evident that the GITT had the capability to solve this problem and capture recirculations, even those of low intensity. As a sequence to this work, charts that cover an appreciable range of variations of Rayleigh number for different leaning angles and aspect ratios are obtained.

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Further reading

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